

DEFINING LOGICAL CONSTANTS. Quine begins with a familiar idea that logical connectives in a first-order language may be reduced to just three (if, as Quine notes, we use the Sheffer stroke, then just two): \sim , \rightarrow , \forall . We have seen, albeit informally, how this can be done. 259
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Quine then turns to the question of determining meaning through definition. This is a question related not to every aspect of meaning (such as its ‘connotation’), but only to ‘truth-conditional meaning’. 260:2

Remark 1. Those who have taken the Philosophy of Language course should consult here the Fregean distinction between ‘sense’ and ‘tone’ that is parallel here to Quine’s distinction between ‘meaning’ and ‘connotation’.

Quine assumes that to specify the meaning of a word W we have to specify the conditions under which the sentence containing it is true. He then considers two ways of meaning determination for W . First, there we may state ‘absolutely’ when the sentence containing W is true. Second, we may do that by ‘relating’ the truth value of the sentence containing W to the truth values of other sentences. Although in definitions we usually determine the meaning of a word relative to other words, we could still try the absolute way. 260:2

Example 2 (Absolute and relative). Suppose we treat the sign ‘ \rightarrow ’ as a meaningless mark. When we draw a truth table for it, we give ‘ \rightarrow ’ absolute meaning independent of the meaning of other words. When we define the sign via other primitives, then we set its meaning relative to the meaning of other words. 260:2

In general, then, we can treat our logical primitives as meaningless marks and then go through the statements containing them arbitrarily assigning truth values to them. In this way we can make them true by convention (or false by convention, but, as Quine observes, there is no need for a special treatment of falsehood). 260:3
261:2

Given this procedure, we can say that the statement S'' in which only logical primitives occur essentially (see Handout 2). Thus the whole of logic may be declared to be true by convention. 261:1

There is a slight complication, in that still there are statements which, in addition to logical primitives, have non-logical expressions occurring essentially in them. They are part of the traditional use of logical primitives (the same sort of traditional use that we encountered earlier in Section I). So we have to furnish additional conventions to fix the meaning of connectives unambiguously. But this problem is not an issue: since the conventions we have already determined are supposed to conform to the traditional use, the meaning of primitives will not be in doubt given these additional statements. 261:2

INFINITY. We have made no assumptions about the number of conventions we have to provide. But consider: if we define primitives by convention in the way described above, can we accomplish this when the conventions have to fix the meaning of infinitely many statements? 261:3

We cannot set conventions individually, one by one, because this would involve in accomplishing an infinity of tasks. Here, in so many words, with an amazing under-statement, Quine actually formulates one of the two main arguments of the whole paper! He will return to it, again very briefly, in page 272. 261:3

To address this infinity problem, we have to settle for conventionalising a finite set of statements that will, in some way, ‘determine’ infinitely many truths. To preview the argument developed much later in pages 270–272, this determination itself relies on a further convention, which in turn requires another convention, and so on. But let us proceed slowly. 262:1

CONVENTIONS FOR PROPOSITIONAL CALCULUS. To generate infinitely many truths from finitely many conventions we now use axioms of propositional calculus, finite in number (in fact, just three), to generate infinitely many formulae. For specificity, Quine uses Łukasiewicz’s postulates in (3-1) which only require as an inference rule *modus ponens* in (3-2).

- (3-1) a. Convention (I): $(p \rightarrow q) \rightarrow ((q \rightarrow r) \rightarrow (p \rightarrow r))$
 b. Convention (II): $p \rightarrow (\sim p \rightarrow q)$
 c. Convention (III): $(\sim p \rightarrow p) \rightarrow p$

(3-2) Convention (II)

$$\begin{array}{l} p \rightarrow q \\ p \\ \therefore q \end{array}$$

Consider (3-1a). It conforms to ordinary usage. Hence we can lay it down as Convention (I). We can then introduce a further Convention (II) corresponding to *modus ponens*. This gives a further infinity of truths. 262:2
262:3

Let us explain this further. The axiom (3-1a) is of the form $\lceil A \rightarrow B \rceil$. So we can suppose that A has the form of (3-1a). Then A is true by convention (I). Then by *modus ponens* (i.e. convention (II)) we can infer B . What B is is undetermined. Still, we can argue that it is true. The reason is simply that *modus ponens* is truth-preserving (that is, sound).

Hence we can get another infinite stock of truths from B and *modus ponens*. 263:1

Now we can add two more conventions (3-1b) and (3-1c). It turns out, as Quine observes, that some statements lacking negation can only be derived by using these latter conventions rather than (3-1a) alone. 263:3
264:2

Question 3. Explain the derivations in 264:2.

It is clear that the only statements derived from (3-1) and (3-2) contain only the signs for implication and negation essentially. This follows from the fact that the initial stock contains only these two signs essentially, and that *modus ponens* does not introduce any additional essential signs. 264:3

Because the axiom system based on (3-1)–(3-2) is ‘complete’, all statements involving only negation and implication essentially are derivable in it. And since other propositional connectives are definable through implication and negation, all statements of propositional calculus can be generated by our conventions. 265:2

PREDICATE CALCULUS. A similar procedure is available for the convention for the use of the universal quantifier. But the details are (fortunately) unimportant, since the procedure is analogous to the propositional case. The upshot is that all logic thus becomes true by convention. 265:3

CONSISTENCY. Quine briefly addresses the problem of consistency (mentioned earlier in page 260). The objection raised by some observers, notably Poincaré, is that any introduction of conventions is subject to a further requirement of consistency which itself cannot be conventionalised away. In the case of piecemeal conventions, what is this demand of consistency in practice? That we do not assign truth both to A and $\sim A$ in the process of laying down conventions. Of course we won’t do any such thing, since this is against the traditional use of negation. 265:4

The issue is slightly more complex with general conventions like (3-1) and (3-2). Here we rely on the fact that *modus ponens* is truth-preserving in accordance with its traditional use. 266:2

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