

Metaphysics // Fall 2017

Handout 12

Mathematical existence: Yablo

THE QUINEAN INSPIRATION. Some ontological disputes can be *resolved*. Problematic ontological claims can be paraphrased away, banned, or grudgingly accepted.

Some ontological disputes can be *dissolved* if one party makes claims in no serious spirit. That party fails to express ontological commitment.

INSTRUMENTALISM. We might say that we often make as-if claims. Yablo lists three problems with it.

REAL CONTENT. There is something that we *do* assert in those as-if claims. For example, if you are an instrumentalist about mathematics, you still assert something when make claims about number properties. The same problem arises, incidentally, with ordinary fiction, I believe.

CORRECTNESS. There are correct and incorrect as-if claims. Where are the standards of correctness coming from?

PRAGMATISM. Even though you claim you do not assert those as-if claims, it seems that you do. Putnam's ghost is back: what exactly is missing from you as-if assertion to turn them into genuine assertions?

META-FICTIONALISM. The proposal is that in as-if asserting S one endorses the as-if assertion of S . The assertoric content of S is the correctness of as-if asserting S . Again, a number of problems here.

MODALITY. It appears that $2 + 3 = 5$ necessarily so. But the standards of mathematical correctness could have been different. Hence the content of what we assert with the aid of a mathematical sentence are not exactly the standard of its correctness.

CONCERN. When we say:

The number of people voting Democrats has fallen, (12-1)

we may get emotionally involved, quite a bit so. But the content of (12-1), according to meta-fictionalism, is just that it is correct to assert it under standard math. Our concern varies wrt these two contents.

PHENOMENOLOGY. On the face of it, making claims such as (12-1) is not about a mathematical theory.

OBJECT FICTIONALISM Linguistic rules, together with the state of the world, determine the truth value of our statements. So the literal content of a statement S will be the worldly condition just enough to make S true according to linguistic rules. So we have:

$$\text{LitCon}(S) = \log_R(S) \tag{12-2}$$

The same can be done with fiction. We will formulate the condition under which S comes out true within fiction:

$$\text{RealCon}(S) = \log_F(S). \tag{12-3}$$

Thus we get, for example:

$$\text{RealCon}(\text{the number of planets equals nine}) = \text{there are nine planets} \tag{12-4}$$

and, incidentally,

$$\text{RealCon}(\text{there is a world where Obama is a sailor}) = \text{Obama could have been a sailor.} \tag{12-5}$$

INTRA-FICTIONAL ASSERTIONS. A particularly nasty problem arises when we consider statements about the properties of fiction itself. Recall that the real content of ' $\#F$ s is n ' is 'there are n F '. Then suppose I say:

According to the number theory, the number of even numbers is greater than one. (12-6)

This entails that:

There really is more than one even number. (12-7)

That is an affront against nominalism. An analogous problem arises when I begin with the claim:

There really are no even numbers. (12-8)

Question 1. Explain the last claim.

REFLEXIVE FICTIONALISM. The solution to the above problem is to observe that the problematic expressions can be taken literally and fictionally within the same context, even within the same sentence. Consider:

It is an insult to pigs to say that men are pigs. (12-9)

The first occurrence of 'pig' in (12-9) is literal, while the second is fictional (metaphorical). We can now try it with numbers:

The number of numbers is 0. (12-10)

At first, this statement seems self-contradictory: at least there is the number 0! But this conclusion we can now block, because the first occurrence of 'number' will be taken as literal, while the second as figurative. So the statement (12-10) becomes a legitimate claim both for a platonist (it is false) and a nominalist (it is true).

We can say that, when 'number' is taken literally it serves as a thing-represented. When it is taken figuratively it serves as a representational aid. There are then three possibilities:

Applied talk 'The number of planets is nine.' Here 'number' is an expression for a representational aid.

Explicit talk 'There are numbers.' In this assertion, if it is made in a philosophical debate, 'number' is an expression for things-represented.

Self-applied talk 'The number of even numbers is greater than 0.' Suppose this is an assertion made in a philosophical debate. Then the first occurrence of 'number' is an expression for a representational aid, the second occurrence is an expression for things-represented.

Question 2. Consider the statement 'The number seven is greater than the number five' asserted by a mathematician (a high-school teacher perhaps). How does the expression 'number' function in it?

RELATIVE REFLEXIVE FICTIONALISM (=FIGURALISM). The limitation of object fictionalism was its lack of flexibility. It could not allow one and the same expression feature as designating a representational aid and a thing represented. Reflexive fictionalism allows for different make-believe games to fix the role of problematic expressions ('X-terms').

Remark 3. For more on make-believe games, see Walton, 'Metaphor and prop oriented make-believe', or his book *Mimesis and Make-Believe*.

The problem with reflexive fictionalism, as formulated so far, is that it does not provide for a possibility of a communication, a debate, between nominalists and platonists. To be sure, it alleviates the unacceptable inference resulting in (12-7), but why would a nominalist bother with it? He may be stubborn enough to not go beyond the claim that there are no numbers. Hence he may be uninterested in finding new ways of characterising talk about numbers.

On the other hand, a platonist does not need the distinction between talking about numbers (in Explicit talk) and not really talking about them (as in Applied talk). He can take his quantification over numbers always at face value. Hence the machinery of fictionalism is for him at most a matter of curiosity.

The new proposal now is to distinguish between saying how things are and saying how things are *as* imagined by others. Consider the statement:

There are at least two prime numbers. (12-11)

If you are a nominalist, you might reject this statement out of hand: there are no numbers in the first place. Then you would speak in a *disengaged* manner. Presumably this would be the case in a philosophical debate with the platonist, and there is not much you can say to each other.

Or you could try to *engage* with number theorists and examine the statement as it appears in their discourse. Then you would in fact agree with that statement.

Yablo further argues that communication among mathematicians, who happen to hold opposing ontological beliefs, has not so far been explained. In particular, it has not been explained by Carnap. The disagreement between philosophically minded mathematicians was simply ignored. This is a puzzling claim. You might have thought that Carnap intended to provide a blueprint for interpreting precisely such disagreements, based on the distinction between external and internal questions. What Yablo should have said, I think, is that the internal/external distinction can only be made sense of by fictionalism. A more careful observation of the ontological exchange would give us precisely the fictionalist model of communication.

According to the figuralist, a nominalist pretends to engage in a game of mathematical assertions. This means *not* that he does not assert mathematical statements—he does. It only means that these statements, if taken literally, have for him a 'problematic' veridical content. Either this content does not exist, so that these statements taken literally are neither true nor false, or else this content is trivially false and in any case *is not asserted* by the nominalist. I am not sure which way Yablo prefers to go (see below).

The platonist, on the other hand, ascribes to those statements both veridical and assertional content.

METAPHORS. The analogy, then, is with metaphors. The nominalist interprets the mathematical discourse, or any other problematic X-discourse, as one might interpret a metaphor. Consider:

Juliet is the sun,

(12-12)

as uttered by Romeo.

Remark 4. The full text is:

It is the east, and Juliet is the sun.
Arise, fair sun, and kill the envious moon,
Who is already sick and pale with grief,
That thou, her maid, art far more fair than she.

Romeo does not assert, literally, that Juliet is the sun, i.e. a mass of helium and hydrogen billions of years old. But he asserts something, namely the real content—presumably some long disjunction of the girl Juliet's characteristics. There is a disagreement in the literature on metaphors whether Romeo says something trivially false (that Juliet consists of helium) and something non-trivially true (that Juliet is central to his life etc.) at the same time. Perhaps he says, but does not assert, that Juliet consists of helium in the first place, or perhaps he does not say that in the first place.

The statement (12-11) will have a similar status in the eyes of a nominalist. There is a level of literal content that is not asserted (or perhaps there is no such *content* at all). And there is a level of figurative content, the real content, that is asserted, and that is a logicist paraphrase of the statement.

YSB