

### Mathematical existence: Putnam

**THE FAILURE OF NOMINALISM.** Nominalists have the ambition to eliminate reference to abstract entities from the discourse of mathematics. Putnam aims to show that nominalism is inadequate for empirical science as well.

Consider Newton's law of gravitation which we will assume to describe an objective law governing the behaviour of material bodies;

$$F = \frac{Gm_a m_b}{d^2}. \quad (8-1)$$

On the face of it, this statement is incompatible with nominalism, so far as it commits us to the existence of numbers. This commitment to mathematical entities is the first obstacle. But there is trouble with representing physical entities in a way acceptable to nominalism, such as forces, masses, and distances.

Yet couldn't a nominalist come up with his paraphrase in principle? There has been as yet no translation of a 'measurement statement', such as 'the force  $F$  is  $q$ '. Putnam then gives an argument to the effect that any nominalistic paraphrase requires a universe of infinitely many physical objects.

*Putnam's argument simplified.* The law (8-1) correlates arbitrary magnitudes. So we must be able to say, 'For every distance  $d$ , masses  $m_1$  and  $m_2$ , the force is  $f$ .' The nominalized paraphrase is characterized by the fact that physical objects play the role of mathematical entities. If there are finitely many individuals, then, in the nominalized language, there must be only finitely many ways of, for example, saying:

$$\text{The distance between } a \text{ and } b \text{ is } n \text{ metres.} \quad (8-2)$$

Which means that the paraphrase will fail, unless there is an assumption of infinitely many objects.

**INDISPENSABILITY AND QUEER QUESTIONS.** There is nothing deviant about philosophical claims of existence. The alleged deviance admits only of a *circular* argument.

Secondly, while there may be 'deviant' pure philosophical claims of existence, mixed claims are accepted by all as non-deviant. That is:

$$\exists x(x \text{ is a number})$$

is supposedly deviant, but

$$\exists x(x \text{ is a number} \ \& \ x \text{ is prime})$$

is not deviant. But then ordinary language is not *deductively closed*. Some statements belong to it, but their logical consequences do not. To close it, we have to admit into it exactly the alleged deviant sentences.

**THE FAILURE OF FICTIONALISM.** Putnam notes one bad reason for the rejection of fictionalism: the dogma of verificationism. It is not logically impossible for us to be brains in a vat, or be directed by the malicious demon. The right reason, to cut the story short, is that the reasons directing us to accept a theory *qua* a theory are the very same reasons directing us to accept the existence of mathematical entities.